

## Written Exam Economics August 2016

### **Monetary Policy**

August 19

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

**This exam question consists of 4 pages in total**

Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

### QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

- (i) In the simple cash-in-advance model with only consumption in the utility function, there is no unique optimal monetary policy.
- (ii) In the Lucas “islands” model, an unanticipated aggregate money shock has no real effects as agents have rational expectations.
- (iii) Under a nominal interest-rate targeting procedure, monetary policymaking performed without knowledge of the realizations of current shocks can be improved by using the money stock as an intermediate target whenever money-market shocks are predominant in the economy.

### QUESTION 2:

#### Strict inflation targeting and nominal interest-rate rules

Consider the following model for output and inflation determination in a closed economy:

$$y_t = \theta y_{t-1} - \sigma (i_{t-1} - E_{t-1} \pi_t) + u_t, \quad 0 < \theta < 1, \quad \sigma > 0, \quad (1)$$

$$\pi_t = \pi_{t-1} + \kappa y_t + e_t, \quad \kappa > 0, \quad (2)$$

where  $y_t$  is log of output in period  $t$ ,  $i_t$  is the nominal interest rate (the monetary policy instrument),  $\pi_t$  is the inflation rate,  $u_t$  and  $e_t$  are independent, mean-zero, serially uncorrelated shocks.  $E_j$  is the rational expectations operator conditional on information up to and including period  $j$ . It is assumed that  $\sigma\kappa < 1$ . The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2} E_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^2, \quad 0 < \beta < 1.$$

- (i) Derive the optimal interest rate rule for  $i_t$  as a function of  $\pi_t$  and  $y_t$ . (Hint: Treat  $E_t y_{t+1} \equiv y_{t+1} - u_{t+1}$  as the policy instrument, and solve the maximization problem by dynamic programming treating  $\pi_t$  as the state variable; i.e., we have  $v(\pi_t) = \max_{E_t y_{t+1}} E_t \left\{ -\frac{1}{2} (\pi_{t+1})^2 + \beta v(\pi_{t+1}) \right\}$ . Then find the optimal policy as  $E_t y_{t+1} = -B\pi_t$ , where  $B$  is a parameter to be found, and use (1) and (2) to derive the associated nominal interest rate.)
- (ii) Comment on the coefficient on  $\pi_t$  in the optimal interest rate rule, with special emphasis on how its value affects the stability properties of the model.
- (iii) Discuss how the coefficients on  $\pi_t$  and  $y_t$  in the optimal interest-rate rule depend on the underlying parameters of the model. Discuss in particular what the parameters reveal about the strict inflation-targeting preferences of the central bank.

### QUESTION 3:

#### Rogoff conservativeness in a New-Keynesian Model?

Consider a one-equation variant of a New-Keynesian model of inflation determination:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (1)$$

where  $\pi_t$  is inflation,  $0 < \beta < 1$  is a discount factor,  $E_t$  is the rational expectations operator,  $\kappa > 0$  is a parameter,  $x_t$  is the output gap, and  $e_t$  is a “cost-push” shock that follows the process

$$e_t = \rho e_{t-1} + \hat{e}_t, \quad 0 < \rho < 1,$$

where  $\hat{e}_t$  is a mean-zero i.i.d. disturbance. It is assumed that the monetary policy-maker controls  $x_t$  with the aim of maximizing the discounted sum of

$$U_t = -\frac{\lambda}{2} x_t^2 - \frac{1}{2} \pi_t^2, \quad \lambda > 0. \quad (2)$$

- (i) Show that under discretionary policymaking, optimal policy is characterized by

$$-\lambda x_t = \kappa \pi_t. \quad (3)$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive “cost-push” shock.

- (ii) Assume now that the policymaker can commit to a policy of the form

$$x_t^c = -\omega e_t, \quad (4)$$

where  $\omega$  is a policy-rule parameter. Derive the optimal relationship between  $x_t^c$  and  $\pi_t^c$ . [Hint: Combine (4) with (1) to show that  $\pi_t^c = [\kappa/(1 - \beta\rho)] x_t^c + [1/(1 - \beta\rho)] e_t$  and maximize  $U_t$  w.r.t.  $x_t^c$  subject to this expression for  $\pi_t^c$ .]

- (iii) Discuss, based on the result of (ii), whether appointing a “conservative” policymaker, one characterized by  $\lambda^c < \lambda$ , is beneficial. Comment on whether  $\rho > 0$  is crucial for the answer.